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EXPERIMENTAL ESTIMATION OF CONTACT CONDITIONS IN PROBLEMS OF MECHANICS OF A SOLID DEFORMABLE BODY

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An experimental technique for estimating the conditions acting on the contact surfaces of mutually deformable bodies as applied to geomechanical measurements is considered.

In solving problems of the mechanics of a solid deformable body, conjugation conditions are used to provide continuity of stresses and shifts at the contacts of media with different mechanical properties. Either full cohesion or full slipping are taken as these conditions [1-4].

Selecting this or that condition, in practice one proceeds from a physical analysis of the operation of the specific system for which the problem is to be solved. There are practically no corresponding methods of experimental estimation.

In what follows a method of experimental determination of conjugation conditions on contact surfaces of deformable bodies is formulated as applied to problems of geomechanical measurements by well sensors. To solve the problem formulated, method of optical polarization modeling of the operation of specific sensors by cylindrical models of the same shape using elements made of optically sensitive materials is employed [2, 5, 6].

A metal plate weakened by a round hole which is strengthened by a disk made of a different material and loaded by a system of forces acting in its plane can serve as an example of contact elements. If a gap between the walls of the hole in the plate and the side surface of the disk is absent, this is achieved, e.g., by to a glue layer, then one can say nothing about real contact conditions. As is known from the mechanics of a solid deformable body, the distribution of stresses and deformations in the plate and the disk will depend on the specific conjugation conditions between the contact elements. In solving these problems, the version of full adhesion is usually adopted. However, the experiments conducted by the author showed that the condition of full adhesion does not always hold in this case.

In experiments with plates made of metal, concerete, different rocks, gypsum, wood, and firebrick and flushing elements (disks, cylinders) made of glass and other optically sensitive materials, two types of optical patterns were observed in the latter in polarized light: a) an isochrome of one color (the entire disk field is of the same color with uniform intensity); b) circular concentric fringes. A theoretical analysis showed that the difference in the patterns is caused by different contact conditions acting in the specific plate–disk system.

We consider the two presented cases in more detail.

Case a. It is known from the literature on the optical polarization method [3-5] that with full adhesion the optical pattern in the disk will consist of an isochrome of one order over the entire field observed. We write the conditions of full adhesion [3, 4]:

$$\sigma_{r1} = \sigma_r\,,\quad \tau_{r\theta 1} = \tau_{r\theta}\,,\quad u_{r1} = u_r\,,\quad v_{\theta 1} = v_\theta\,. \tag{1}$$

We present the final formulas for stresses in a solid disk in a polar system of coordinates under the condition of full adhesion

$$\sigma_{r1} = (p + q)\, D/2 + (p - q)\, H\, (\cos 2\theta)/2\,,$$

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$$\sigma_{\theta 1} = (p + q)\, D/2 - (p - q)\, H\,(\cos 2\theta)/2\,, \quad \tau_{r\theta 1} = (p - q)\, H\,(\sin 2\theta)/2\,, \tag{2}$$

where in the case of flat deformation

$$D = 2\,(1 - \nu^2)\, E_1 / [(1 + \nu_1)(1 - 2\nu_1)\, E + (1 + \nu_1)\, E_1]\,,$$
$$H = 4\,(1 - \nu^2)\, E_1 / [(1 + \nu_1)\, E + (3 - 4\nu_1)(1 + \nu)\, E_1]\,. \tag{3}$$

In formulas (1)-(3) σ_{r1}, $\sigma_{\theta 1}$, $\tau_{r\theta 1}$, σ_r, $\tau_{r\theta}$ are the radial, tangential, and shearing stresses, respectively, in the disk material and the plate; u_{r1}, $v_{\theta 1}$, u_r, v_θ are the radial and tangential shifts in the disk material and the plate; p, q are the uniformly distributed forces loading the plate; E_1, ν_1, E, ν are the elasticity modulus and the Poisson coefficient in the disk material and the plate, respectively; r, θ are the polar coordinates.

In accordance with the main law of photoelasticity the difference in the path of light beams Γ in a stressed disk made of optically sensitive material is related to the difference in the principal stresses in them $(\sigma_1 - \sigma_2)$ by the dependence [2, 3]

$$\Gamma = Cd\,(\sigma_1 - \sigma_2) \tag{4}$$

or, written in terms of maximum shearing stresses, by the relation [2, 3]

$$\Gamma = 2Cd\tau_{\max}\,. \tag{5}$$

Here C is the optical constant of the stress sensor material, d is the path of light in the disk, $\tau_{\max}$ is the maximum shearing stress in the disk material, where

$$\tau_{\max} = 0.5\sqrt{(\sigma_{r1} - \sigma_{\theta 1})^2 + 4\tau_{r\theta 1}^2}\,. \tag{6}$$

Substituting $\tau_{\max}$ from (6) into (5) and allowing for (2) and (3), one can easily obtain

$$\Gamma = 2CdH\,(p - q) = 8Cd\,(p - q)(1 - \nu^2)\, E_1 / [(1 - \nu_1)\, E + (3 - 4\nu_1)(1 - \nu)\, E_1]\,. \tag{7}$$

It follows from (7) that the multiplier at $(p - q)$ involves the optical and the elastic constants of the disk material and elastic constants of the plate material (rock, concrete, etc.), and also quantity d. Under the conditions of a specific experiment all the quantities enumerated remain constant and in Eq. (7) the quantity $(p - q)$ is variable during measurements. By virtue of the above we rewrite (7) in the following form

$$\Gamma = K\,(p - q)\,, \tag{8}$$

where K is some constant quantity assigned by (7).

According to (8) the difference in the path of light beams in the disk (sensor) will be the same at all points of the disk (sensor) cross-section, since it does not depend on the coordinates r and θ. Consequently, at the inlet of a solid glass cylindrical sensor monochromatic color should be observed in polarized light, or, in terms of photoelasticity, an isochrome of one order. The theoretical result presented is in agreement with the experimental data.

Case b. In a continuous (without an axial hole) cylindrical glass sensor (disk) an optical pattern of circular coaxial fringes is observed in the experiment. To describe this result analytically, total slippage at the contact of the cylindrical sensor with the well walls (the disk with the hole walls in the plate) is assumed [7]:

$$\sigma_{r1} = \sigma_r\,, \quad u_{r1} = u_r\,, \quad \tau_{r\theta 1} = 0\,, \quad \tau_{r\theta} = 0\,. \tag{9}$$

Without giving the corresponding solution (which is obtained in a standard manner after combersome computations), we write final formulas for stresses in the disk ($r \le R$), R is the sensor radius [7]):

$$\sigma_{r1} = (p - q)\, A/2 + (p - q)\, B\, (\cos 2\theta)/2\,,$$

$$\sigma_{\theta 1} = (p - q)\, A/2 - (p - q)\, B\, (\cos 2\theta)/2\,,$$

$$\tau_{r\theta 1} = (p - q)\, F\, (\sin 2\theta)/2\,, \tag{10}$$

where for the case of plane deformation

$$A = 2\,(1 - \nu^2)\, E_1/[(1 + \nu)\, E_1 + (1 + \nu_1)\,(1 - 2\nu_1)\, E]\,,$$

$$B = 6\,(1 - \nu^2)\, E_1/[(5 - 6\nu)\, E_1 + (1 + \nu_1)\,(3 - 2\nu_1)\, E]\,,$$

$$F = 6\,(1 - \nu^2)\, E_1\,(2r^2/R^2 - 1)/[(5 - 6\nu)\,(1 + \nu)\, E_1 + (1 + \nu_1)\,(3 - 2\nu_1)\, E]\,. \tag{11}$$

In (9)-(11) the notation is the same as in (1)-(3). Substituting τ_{max} from (6) into (5) and taking into account relations (10), (11), we obtain an expression for the difference of the path of light beams in a photoelastic sensor (disk) in the case of total slippage:

$$\Gamma = CdB\,(2r^2/R^2 - 1)\,(p - q) = Cd\,(p - q)\,(2r^2/R^2 - 1) \times$$

$$\times\, 6\,(1 - \nu^2)\, E_1/[(5 - 6\nu)\, E_1 + (1 + \nu_1)\,(3 - 2\nu_1)\, E]\,. \tag{12}$$

In (12) the multiplier at $(p - q)$ depends on the optical constant C, the elastic constants E_1 and ν_1 of the sensor (disk) material, the elastic constants E and ν of the plate (rocks, concrete) material, and the radius R and the length d of the sensor, which are constant during an experiment. Comparing (7) and (12) we see that in the latter relation the multiplier $(2r^2/R^2 - 1)$ appeared, which depends on the coordinate r.

We rewrite (12) in the following form

$$\Gamma = K\,(p - q)\,(2r^2/R^2 - 1)\,, \tag{13}$$

where K is some constant.

From (13) we find

$$r^2 = [\Gamma + K\,(p - q)]/[2K\,(p - q)\, R]\,.$$

In a particular case with the plate and the sensor being loaded by a uniaxial press (the condition of the experiments) $p = 0$, and the quantity q under the conditions of specific loading will be constant. Then

$$r^2 = [\Gamma + \mathrm{M}_1]/\mathrm{M}_2\,, \tag{14}$$

where M_1 and M_2 are constants.

Relation (14) is the equation of a concentric circle with its center at the coordinate origin. Consequently, according to (14) the optical pattern in a solid photoelastic sensor (disk) under the conditions of total slippage at the contact of the sensor with the well walls (the disk with the hole walls in the plate) should consist of concentric circles coaxial with the longitudinal axis of the sensor.

The result presented is in agreement with the test data. From 3-4 to 10 circular isochromes were observed in an experiment with gypsum plates with a glass sensor.

The effect of a glue layer was not taken into account in the results presented. Corresponding experiments showed that, when glues of the type of hardened epoxide resins are used, one isochrome is observed, i.e., the case of full adhesion. Both cases are observed with gypsum or water-cement glues. A theoretical analysis of the effect of a glue layer is practically impossible within the framework of a magazine article.

All the above allows one to suggest a technique for estimating actual contact conditions in geomechanical measurements using cylindrical well sensors. We assume that the sensor body is made of metal (the most common in the practice of geomechanical measurements). It is necessary to estimate experimentally the contact conditions at the boundary between the sensor and the well walls.

It is proposed to act in the following way. We assume that measurements are to be taken in a concrete structure (e.g., in a solid basement, in the body of a concrete dam, etc.). A cube or a plate with a side of 20 cm (the side size should exceed the sensor diameter by 4-5 times) is made of concrete of the same type as the studied structure is made. In the center of the cube (plate) an axial hole is drilled the diameter of which coincides with the diameter of the measuring well (i.e., the well which will be used in full-scale measurements to place the sensor in the concrete structure). A metal cylinder simulating the sensor is positioned in this hole. The gap between the cylinder and the hole walls is filled with the glue that is to be used in the measurements. The dimensions of the metal cylinder should correspond to the sensor and the cylinder should be made of the same metal as the sensor body is made. The cylinder ends are mirror polished and disks of an optically sensitive material are glued to them (photoelastic coating [2, 8]). The concrete cube with a metal cylinder provided with photoelastic coatings (there could be only one coating) is uniaxially loaded by a press.

The optical pattern in the photoelastic coatings is studied during loading. If the optical pattern consists of an isochrome of one order through the entire photoelastic coating, then the contact conditions between the metallic cylinder and the hole walls correspond to the case of full adhesion (1). In the cases when the optical pattern in the photoelastic coatings consists of circular isochromes, then the contact conditions correspond to total slippage (9).

It is obvious that in a real sensor with a cylindrical metal body positioned in a full-scale concrete structure the same contact conditions as in the described experiment will take place.

If the measurements are made in rocks, then the cube (plate) should be made of the same rock in which the measurements are planned.

Another particular case is possible. Circular glass sensors in the form of a cylinder with a small-diameter axial hole are widely used in geomechanical measurements. They are placed in wells and are glued to their walls. The described technique is used to estimate the actual contact conditions of their operation.

Glass sensor models in the form of continuous glass cylinders were produced. The latter were placed in specially drilled holes in the centers of rock plates. In this case the same glues as in the full-scale measurements were used to glue them to the hole walls. The plates with the sensor models were loaded by uniaxial presses. The actual contact conditions were judged from the shape of the optical pattern.

The conducted studies made it possible to establish two types of the above-described optical pattern. Correspondingly, two different theories had to be developed to decode the readings of circular well sensors: one for the case of total adhesion at the contact between the sensor and the well walls, the other for the conditions of total slippage at the contact.

The described technique in the particular case of cylindrical filling elements allows one to estimate experimentally the actual contact conditions formed during operation of a real structure at the contact of the filling element with the surrounding material.

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